Reply by Author to B.T. Fang

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THE justification for the Technical Note[1] "Practical Aspect of the Generalized Inverse of a Matrix" lies in the fact that the constraint defined by Eq. (6) leads to Eq. (10), of which the latter defines a form of the generalized inverse. It is recognized that the column matrix { α } in Eq. (7) of Ref. 1 corresponds to the column of Lagrangian multipliers associated with the minimum norm approach. As a practicing aeroelastician, this author occasionally needs a closed form expression for a solution to simultaneous equations with more unknowns than equations. In his opinion, the insight provided by Eq. (6), etc. of Ref. 1 is important to the aeroelastician. As it happens, the accompanying background and development considered necessary for providing this insight, summarize the most important aspects of the generalized inverse from an aeroelastician's point of view. It seems that the resulting restrictions and repetition of generally available results are the main target of Mr. Fang's remarks.

References

[1]Hassig, H. J., "Practical Aspect of the Generalized Inverse of a Matrix," *AIAA Journal*, Vol. 13, November 1975, pp. 1530-1531.

Received May 5, 1976.
Index category: Aeroelasticity and Hydroelasticity.
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Supersonic Interference Flow Effects on Finned Bodies

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IN a series of publications,[1-4] Korkegi has dealt with many aspects of three-dimensional shock wave boundary-layer interactions including the greater susceptibility to separation than for the two-dimensional case, and the effect of boundary-layer transition which can cause a dramatic change of the 'corner flow' interaction between a fin and a fuselage. When one considers this in the light of the observed large crossflow effects on boundary-layer transition on slender vehicles,[5,6] one realizes how dominating an influence this flow interference can have on the rigid and elastic vehicle dynamics of a finned missile. The following comments delineate some of the possibilities.

During a recent hypersonic aeroelastic analysis of tactical missiles,[7] it was found that the wedge-shaped fin, which is most efficient from two-dimensional flow considerations (Fig. 1), could be dynamically unstable for rotation axis locations as far forward at 20% chord (Fig. 2). The experimental data showing this were obtained by East.[8] Here, the "corner flow" is located between a wedge and the endplate used to prevent the tunnel wall boundary layer from interfering with the two-dimensional airfoil test. One can, of course, expect the interference to be much stronger for a fin

Received March 15, 1976; revision received May 27, 1976.
Index categories: Nonsteady Aerodynamics; Supersonic and Hypersonic Flow.
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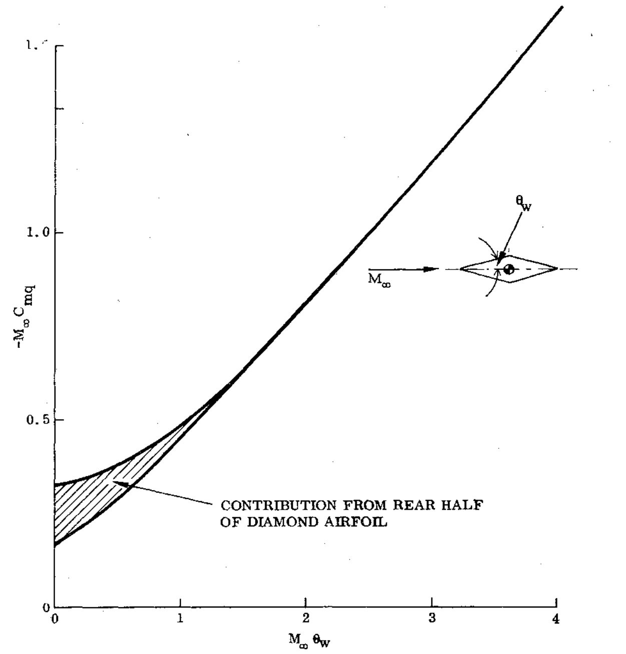

Fig. 1 Damping in pitch of double-wedge airfoils as a function of Mach number and wedge angle.

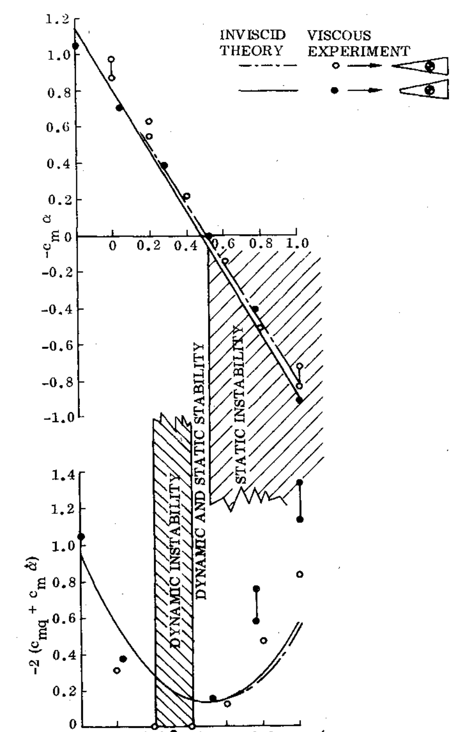

Fig. 2 Theoretical and experimental unsteady aerodynamic characteristics of a 9.5° wedge as a function of rotation axis location at $M_\infty = 9.7$.

mounted on a missile fuselage. Korkegi[3] has shown that at high supersonic speed, any reasonable wedge angle will cause separation of the boundary layer on a flat sidewall, and consequently, separation of the crossflow-weakened boundary layer on a missile body.

Korkegi[2,4] discusses how the extensive sidewall boundary-layer separation existing for laminar flow conditions can be drastically reduced and even eliminated in the presence of boundary-layer transition. Ward[9] has shown that at crossflow angles as low as 20% of the cone half angle, the

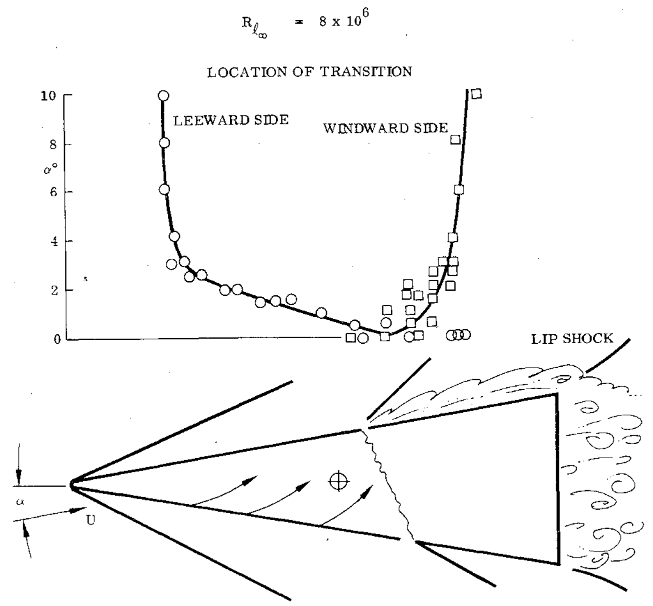

Fig. 3 Effect of angle of attack on boundary-layer transition on a 10° sharp cone at $M_\infty = 6$.

leeward side boundary-layer transition on a sharp 10° cone moves half a body length forward (of the windward and meridian side transition fronts). (See Fig. 3.) On a 5° cone, this large movement would occur already at $\alpha = 1°$, as α/Θ_c is the relevant crossflow parameter,[5] and at a mere $\alpha = 0.5°$ if the conic frustrum has a moderate amount of boundary-layer mass addition (through ablation, for example). On an ogive-cylinder, the typical missile body, the required crossflow angle is likely to be even less. A sudden change of separated flow pattern usually causes highly nonlinear (often discontinuous) changes of the aerodynamic characteristics, with accompanying extremely large effects on elastic and rigid body dynamics.[10-12] Thus, much analytic and experimental work is needed before one can design a finned missile that will work successfully at high supersonic and hypersonic speeds.

References

[1] Korkegi, R. H., "On the Structure of Three-Dimensional Shock-Induced Separated Flow Regions," AIAA Paper 76-165, Washington, D.C., 1976.

[2] Korkegi, R. H., "Survey of Viscous Interactions Associated with High Mach Number Flight," *AIAA Journal,* Vol. 9, May 1971, pp. 771-784.

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[4] Korkegi, R. H., "Comparison of Shock-Induced Two- and Three-Dimensional Incipient Turbulent Separation," *AIAA Journal,* Vol. 13, April 1975, pp. 534-535.

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[6] Ericsson, L. E., "Transition Effects on Slender Vehicle Stability and Trim Characteristics," *Journal of Spacecraft and Rockets,* Vol. 11, Jan. 1974, pp. 3-11.

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[10] Ericsson, L. E., "Aeroelastic Instability Caused by Slender Payloads," *Journal of Spacecraft and Rockets,* Vol. 4, Jan. 1967, pp. 65-73.

[11] Ericsson, L. E., "Unsteady Aerodynamics of Separating and Reattaching Flow on Bodies of Revolution," Recent Research on Unsteady Boundary Layers, Vol. 1, IUTAM Symposium, Laval University, Quebec, 24-28 May 1971, pp. 481-512.

[12] Reding, J. P. and Ericsson, L. E., "Delta Wing Separation Can Dominate Shuttle Dynamics," *Journal of Spacecraft and Rockets,* Vol. 10, July 1973, pp. 421-428.

Errata

Hemisphere-Cylinder in Transonic Flow, $M_\infty = 0.7 \simeq 1.0$

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[AIAA J, **13,** 1411-1413 (1975)]

ON page 1412, the first line of Eq. (1) should read:

$$\left(1-\frac{u^2}{a^2}\right)\frac{1}{\kappa}\frac{\partial}{\partial s}\left(\frac{1}{\kappa}\frac{\partial \phi}{\partial s}\right)$$

Received April 27, 1976.
Index categories: Subsonic and Transonic Flow; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

Radiation from an Array of Longitudinal Fins of Triangular Profiles

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[AIAA J. **13,** 691-693 (1975)]

THE abscissa for Fig. 4 and 5 must be corrected by multiplying the values of N_C by $(1/N_L)^2$. For example if $\epsilon = 1.0$, $N_f = 4$, $N_L = 8$ and $Q^* = 4.0$, the corresponding value of N_C from Fig. 5 is approximately 1.5. This should be changed to $N_C = 1.5\,(1/8)^2 = .0234$.

Received May 10, 1976.
Index category: Radiation and Radiative Heat Transfer.